MATHEMATICS

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AIM POINTS
MATHEMATICS
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(MAIN + ADVANCE) & COMPETITIVE EXAM.
FOR XI (PQRS)

GEOMETRIC PROGRESSIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. A sequence of non–zero numbers is called a geometric progression if the ratio of a term and the term preceding to it is always a constant quantity.

   The constant ratio is called the common ratio of the G.P.

2. If $a_1, a_2, a_3, ..., a_n, ...$ is a G.P., then the expressin.

   $a_1 + a_2 + a_3 + ... + a_n + ...$

   is called a geometric series.

3. The nth term of a G.P. with first term 'a' and common ratio 'r' is given by $a_n = a\,r^{n-1}$.

4. If a G.P. consists of m terms, then n^{th} term from the end is $(m - n + 1)^{th}$ term from the beginning and is given by ar^{m-n}.

   If l is the last term of a G.P., the nth term from the end is given by $l\left(\frac{1}{r}\right)^{n-1}$.

5. In a G.P., the product of the terms equidistant from the beginning and the end is always same and is equal to the product of first and last term.

6. If sum of n terms of a G.P. with first term 'a' and common ratio is given by

   $$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

   or, $$S_n = a\left(\frac{1 - r^n}{1 - r}\right), \text{ if } r \neq 1$$

   $$S_n = n, \text{ if } r = 1$$

   $$S_n = \frac{1 - lr}{1 - r}$$

   or, $$S_n = \frac{lr - a}{r - 1}$$

   where l is the last term.

7. If all the terms of G.P. be multiplied or divided by the same non–zero constant, then it remains a G.P. with the same common ration.

8. The rdciprocals of the terms of a given G.P. form a G.P.

9. If each term of a G.P. be raised to the same power the resulting sequence also forms a G.P.

10. Three numbers a, b, c are in G.P. iff $b^2 = ac$.

    If a, b, c are in G.P., then b is known as the geometric mean of a and c.

11. If the terms of a given G.P. are chosen at regular intervals, then the new sequence so formed also forms a G.P.

12. If geometric mean of a and b is given by $\sqrt{ab}$

13. If n geometric means are inserted between two quantities, then the product of n geometric means is n^{th} power of the single geometric mean between the two quantities.
14. If AM and GM between two numbers are in the ration m : n, then the numbers are in the ratio $m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$.

EXERCISE-1

1. Show that the sequence given by $a_n = 3(2^n)$, for all $n \in N$, is a G.P. Also, find its common ration.
2. Prove that the nth term of a G.P. with first term a and common ratio r is given by $a_n = ar^{n-1}$.
3. Prove that the nth term from the end of a G.P. with last term l and common ratio r is given by

$$a_b = l\left(\frac{1}{r}\right)^{n-1}$$

4. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the xommon ratio of the G.P.
5. If the pth, qth and rth terms of a G.P. are a, b, c respectively, prove that :
$a^{(q-r)}, b^{(r-p)}, c^{(p-q)} = 1.$
6. If a, b, c are respectively the pth, qth and rth terms of a G.P., show that :
$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0.$
7. If pth, qth and rth terms of an A.P. as well as a G.P. are a, b and c respectively.
Prove that $a^{b-c} b^{c-a} c^{a-b} = 1$.
8. Find all sequences which are simultaneously A.P. and G.P.
9. Show that the products of the corresponding terms of the sequences a, ar, ar^2, ..., ar^{n-1} and A, AR, AR^2, ..., AR^{n-1} form a G.P., and find the common ratio.
10. The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of the G.P.
11. Find :
(i) the 8th term of the G.P. 0.3, 0.06, 0.012, ...
(ii) nth term of the G.P. $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \ldots$
12. Which term of the progression 0.004, 0.02, 0.1, ... is 12.5 ?
13. Find the 4th term from the end of the G.P.

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots, \frac{1}{4374}$$

14. The fourth term of a G.P. is 27 and 7th term is 729, find the G.P.
15. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.
16. Find three numbers in G.P. whsoe sum is 13 and the sum of whose squares is 91.
17. Three numbers are in G.P. whsoe sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

18. Find four numbers in G.P. in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18.
19. Find three numbers in G.P. whose sum is 38 and their product is 1728.
20. Find the sum of the series 2 + 6 + 18 + ... + 4374.
21. Find the sum of n terms of the sequence $< a_n >$ given by $a_n = 2^n + 3n, n \in N$
22. Determine the number of terms in G.P. $< a_n >$, if $a_1 = 3$, $a_n = 96$ and $S_n = 189$.
23. The sum of some terms of a G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.
24. If S be the sum, P the product and R the sum of the reciprocals of n terms of a G.P., prove that : $\left(\frac{S}{R}\right)^n = p^2$.
25. If S_1, S_2, S_3 be respectively the sums of n, 2n, 3n terms of a G.P., then prove that $S_1^2 S_2^2 = S_1(S_2 + S_3)$.
26. If $S_1, S_2, ..., S_n$ are the sums of n terms of n G.P.'s whose first term is 1 in each and common ratios are 1, 2, 3, ..., n respectively, then prove that $S_1 + S_2 + 2S_3 + 3S_4 + ... (n-1) S_n = 1^n + 2^n + 3^n + ... + n^n$.
27. Find the sum of 2n terms of the series whose every even term is 'a' times the term before it and every odd terms is 'c' times the term before it, the first term being unity.
28. Find the sum of infinity of the G.P. $-\frac{5}{4}, \frac{5}{16}, -\frac{5}{64}, ...$
29. If $a + a^2 + a^3 + ... \infty$, prove that $a = \frac{b}{1+b}$.
30. If $x = 1 + a + a^2 + ... \infty$, where $| a | < 1$ and $y = 1 + b + b^2 + ... \infty$, where $| b | < 1$. Prove that :

$$1 + ab + a^2b^2 + ... \infty = \frac{xy}{x+y-1}$$

31. The sum of an infinite G.P. is 57 and the sum of their cubes is 9747, find the G.P.
32. If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.
33. A square is drawn by joining the mid–points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the square is 10 cm, find the sum of the areas of all the squares so formed.
34. If S_p denotes the sum of the series $1 + r^p + r^{2p} + ...$ to ∞ and s_p the sum of the series $1 - r^p + r^{2p} - ...$ to ∞, prove that $S_p + s_p = 2 . S_{2p}$.
35. Express the recurring decimal 0.125125125 ... as a rational number.
36. IfS denotes the sum of an infinite G.P. S_1 denotes the sum of the squares of its terms, then prove that the first term and common ratio are respectively $\frac{2SS_1}{S^2 + S_1}$ and $\frac{S^2 - S_1}{S^2 + S_1}$.
37. If all the terms of a G.P. be multiplied or divided by the same non–zero constant, then it remains a G.P. with

the same common ratio.

38. There non–zero numbers a, b, c are in G.P. iff $b^2 = ac$.
39. If a, b, c are in A.P. and x, y, z are in G.P., then show that $x^{b-c} . y^{c-a} . z^{a-b} = 1$.
40. If a, b, c are in G.P. and x, y are the arithmetic means of a, b and b, c respectively, then prove that :

$$\frac{a}{x}+\frac{c}{y}=2 \text{ and } \frac{1}{x}+\frac{1}{y}=\frac{2}{b}$$

41. If a, b, c, d are in G.P., prove that a + b, b + c, c + d are also in G.P.
42. If $a^2 + b^2$, ab + bc and $b^2 + c^2$ are in G.P., prove that a, b, c are in G.P.
43. If a, b, c are in G.P., prove that :

$$\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m} \text{ are in A.P.}$$

44. If $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are three consecutive terms of an A.P., prove that a, b, c are the three consecutive terms of a G.P.
45. If a, b, c are in G.P., prove that the following are also in G.P. :
    (i) $a^2, b^2 c^2$ (ii) a^3, b^3, c^3
    (iii) $a^2 + b^2, ab + bc, b^2 + c^2$
46. If (a – b), (b – c), (c – a) are in G.P., then prove that $(a + b + c)^2 = 3(ab + bc + ca)$.
47. If a, b, c are in A.P. and a, b, d are in G.P., show that a, (a – b), (d – c) are in G.P.
48. If a, b, c are three distinct real numbers in G.P. and a + b + c = xb, then prove that either $x < -1$ or $x > 3$.
49. If n geometric means are inserted between two quantities, then the product of n geometric means is the nth power of the single geometric mean between the two quantities.
50. If A and G are respectively arithmetic and geometric means between two positive quantities a and b, then the quadratic equation having a, b as its roots is $x^2 - 2Ax + G^2 = 0$.
51. If A and G be the A.M. and G.M. between two positive numbers, then the numbers are $A \pm \sqrt{A^2 - G^2}$.
52. Find two positive numbers whose difference is 12 and whose A.M. exceeds the G.M. by 2.
53. Find two numbers whose arithmetic mean is 34 and the geometric mean is 16.
54. If the A.M. and G.M. between two numbers are in the ratio m : n, then prove that the numbers are in the ratio $m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$.
55. If a is the A.M. of b and c and the two geometric means are G_1 and G_2, then prove that $G_1^3 G_2^3 = 2$ abc.
56. If one A.M., A and two geometric means G_1 and G_2 inserted between any two positive numbers, show that $\frac{G_2^1}{G} + \frac{G_2^2}{G_1} = 2A$.

EXERCISE-2

Answer each of the following questions in one word or one sentence of as per exact requirement of the questions :

1. If $a = 1 + b + b^2 + b^3 + \ldots$ to ∞, then write b in terms of a.

EXERCISE-3

Mark the correct alternative in each of the following

1. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then xyz are in

(a) AP (b) GP (c) HP (d) none of these

2. The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 92. The common ratio of the original G.P. is

(a) 1/2 (b) 2/3 (c) 1/3 (d) –1/2

3. If a, b, c are in G.P. and x, y are AM's between a, b and b, c respectively, then

(a) $\frac{1}{x}+\frac{1}{y}=2$ (b) $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$ (c) $\frac{1}{x}+\frac{1}{y}=\frac{2}{a}$ (d) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$

4. Given that x > 0, the sim $\sum_{n=1}^{\infty}\left(\frac{x}{x+1}\right)^{n-1}$ equals :

(a) x (b) x + 1 (c) $\frac{x}{2x+1}$ (d) $\frac{x+1}{2x+1}$

5. In a G.P. of even number of terms, the sum of all terms is give times the sum of the odd terms. The common ratio of the G.P. is

(a) $-\frac{4}{5}$ (b) $\frac{1}{5}$ (c) 4 (d) none of these